

# 1. The Physics of Quantum Information: Basic Concepts

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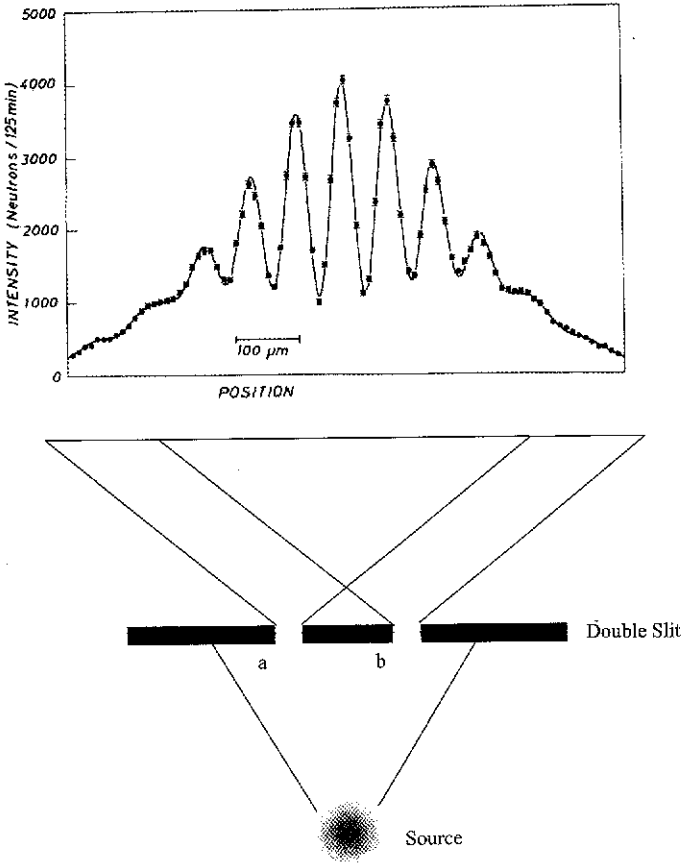
## 1.1 Quantum Superposition

The superposition principle plays the most central role in all considerations of quantum information, and in most of the “gedanken” experiments and even the paradoxes of quantum mechanics. Instead of studying it theoretically or defining it abstractly, we will discuss here the quintessential experiment on quantum superposition, the double-slit experiment (Fig. 1.1). According to Feynman [1], the double-slit “has in it the heart of quantum mechanics”. The essential ingredients of the experiment are a source, a double-slit assembly, and an observation screen on which we observe interference fringes. These interference fringes may easily be understood on the basis of assuming a wave property of the particles emerging from the source. It might be mentioned here that the double-slit experiment has been performed with many different kinds of particles ranging from photons [2], via electrons [3], to neutrons [4] and atoms [5]. Quantum mechanically, the state is the coherent superposition

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\Psi_a\rangle + |\Psi_b\rangle), \quad (1.1)$$

where  $|\Psi_a\rangle$  and  $|\Psi_b\rangle$  describe the quantum state with only slit  $a$  or slit  $b$  open.

The interesting feature in the quantum double-slit experiment is the observation that, as confirmed by all experiments to date, the interference pattern can be collected one by one, that is, by having such a low intensity that only one particle interferes with itself. If this happens, we might be tempted to ask ourselves which of the two slits a particle “really” takes in the experiment. The answer from standard quantum mechanics is that it is not possible to make any sensible statement about the question “which slit does the particle pass through?” without using the appropriate set-up able to answer that question. In fact, if we were to perform any kind of experiment determining through which of the two slits the particle passes, we would have to somehow interact with the particle and this would lead to decoherence, that is, loss of interference. Only when there is no way of knowing, not even in principle,



**Fig. 1.1.** Principle of the double-slit experiment. An interference pattern arises in an observation plane behind a double-slit assembly, even if the intensity of the source is so low that there is only one particle at a time in the apparatus. The actual interference pattern shown here is the experimental data obtained for a double-slit experiments with neutrons [4].

through which slit the particle passes, do we observe interference. As a small warning we might mention that it is not even possible to say that the particle passes through both slits at the same time, although this is a position often held. The problem here is that, on the one hand, this is a contradictory sentence because a particle is a localised entity, and, on the other hand, there is no operational meaning in such a statement. We also note that one can have partial knowledge of the slit the particle passes at the expense of partial decoherence.

## 1.2 Qubits

The most fundamental entity in information science is the bit. This is a system which carries two possible values, “0” and “1”. In its classical realisation the bit, which, for example could be imagined to be just a mechanical switch, is a system which is designed to have two distinguishable states; there should be a sufficiently large energy barrier between them that no spontaneous transition, which would evidently be detrimental, can occur between the two states.

The quantum analog of a bit, the *Qubit* [6], therefore also has to be a two-state system where the two states are simply called  $|0\rangle$  and  $|1\rangle$ . Basically any quantum system which has at least two states can serve as a qubit, and there are a great variety possible, many of which have already been realised experimentally. The most essential property of quantum states when used to encode bits is the possibility of coherence and superposition, the general state being

$$|Q\rangle = \alpha|0\rangle + \beta|1\rangle, \quad (1.2)$$

with  $|\alpha|^2 + |\beta|^2 = 1$ . What this means is not that the value of a qubit is somewhere between “0” and “1”, but rather that the qubit is in a superposition of both states and, if we measure the qubit we will find it with probability  $|\alpha|^2$  to carry the value “0” and with probability  $|\beta|^2$  to carry the value “1”;

$$p(\text{“0”}) = |\alpha|^2, \quad p(\text{“1”}) = |\beta|^2. \quad (1.3)$$

While by the definition of the qubit we seem to lose certainty about its properties, it is important to know that (1.2) describes a *coherent* superposition rather than an incoherent mixture between “0” and “1”. The essential point here is that for a coherent superposition there is always a basis in which the value of the qubit is well defined, while for an incoherent mixture it is a mixture whatever way we choose to describe it. For simplicity consider the specific state

$$|Q'\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \quad (1.4)$$

This clearly means that with 50% probability the qubit will be found to be either in “0” or “1”. But interestingly, in a basis rotated by  $45^\circ$  in Hilbert space the value of the qubit is well-defined. We might simply study this by applying the proper transformation to the qubit. One of the most basic transformations in quantum information science is the so-called Hadamard transformation whose actions on a qubit are

$$H|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad H|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (1.5)$$

Applying this to the qubit  $|Q'\rangle$  above, results in

$$H|Q'\rangle = |0\rangle. \quad (1.6)$$

that is, a well-defined value of the qubit. This is never possible with an incoherent mixture.

### 1.3 Single-Qubit Transformations

Insight in some of the most basic experimental procedures in quantum information physics can be gained by investigating the action of a simple 50/50 beamsplitter. Such beamsplitters have been realised for many different types of particles, not only for photons. For a general beamsplitter, as shown in Fig. 1.2, let us investigate the case of just two incoming modes and two outgoing modes which are arranged as shown in the figure.

For a 50/50 beamsplitter, a particle incident either from above or from below has the same probability of 50% of emerging in either output beam, above or below. Then quantum unitarity, that is, the requirement that no particles are lost if the beamsplitter is non-absorbing, implies certain phase conditions on the action of the beamsplitter [7] with one free phase. A very simple way to describe the action of a beamsplitter is to fix the phase relations such that the beamsplitter is described by the Hadamard transformation of (1.5).

Let us again assume that the incident state is the general qubit

$$|Q\rangle_{in} = \alpha|0\rangle_{in} + \beta|1\rangle_{in}. \quad (1.7)$$

For a single incident particle this means that  $\alpha$  is the probability amplitude to find the particle incident from above and  $\beta$  is the probability amplitude for

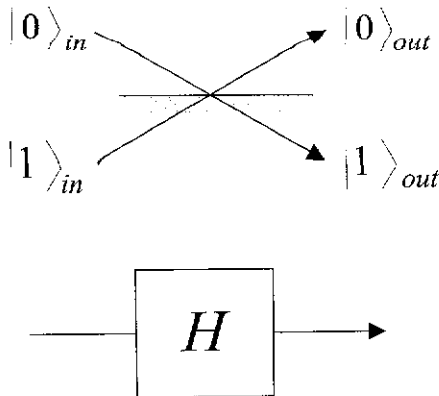


Fig. 1.2. The 50/50 beamsplitter (top) and the corresponding diagram using the Hadamard transform  $H$  (below).

finding the particle incident from below. Then the action of the beamsplitter results in the final state

$$|Q\rangle_{out} = H|Q\rangle_{in} = \frac{1}{\sqrt{2}} ((\alpha + \beta)|0\rangle_{out} + (\alpha - \beta)|1\rangle_{out}) , \quad (1.8)$$

where  $(\alpha + \beta)$  is now the probability amplitude for finding the particle in the outgoing upper beam and  $(\alpha - \beta)$  is the probability amplitude for finding it in the outgoing lower beam. For the specific case of either  $\alpha = 0$  or  $\beta = 0$ , we find that the particle will be found with equal probability in either of the outgoing beams. For another specific case,  $\alpha = \beta$ , we find that the particle will definitely be found in the upper beam and never in the lower beam.

It is interesting and instructive to consider sequences of such beamsplitters because they realise sequences of Hadamard transformations. For two successive transformations the Mach-Zehnder interferometer (Fig. 1.3) with two identical beamsplitters results.

Furthermore, the mirrors shown only serve to redirect the beams; they are assumed to have identical action on the two beams and therefore can be omitted in the analysis. The full action of the interferometer can now simply be described as two successive Hadamard transformations acting on the general incoming state of (1.7):

$$|Q\rangle_{out} = HH|Q\rangle_{in} = |Q\rangle_{in} . \quad (1.9)$$

This results from the simple fact that double application of the Hadamard transformation of (1.5) is the identity operation. It means that the Mach-Zehnder interferometer as sketched in Fig. 1.3, with beamsplitters realising the Hadamard transformation at its output, reproduces a state identical to the input. Let us consider again the extreme case where the input consists of one beam only, that is, without loss of generality, let us assume  $\alpha = 1$ , the lower beam being empty. Then, according to (1.9), the particle will def-

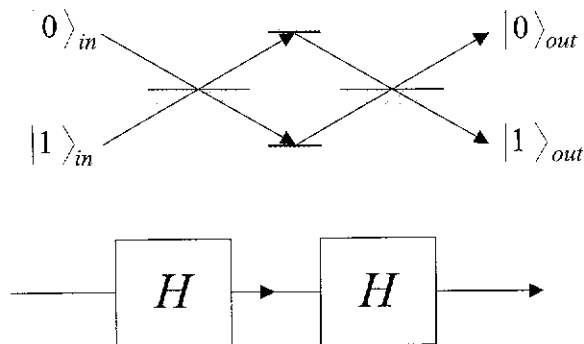


Fig. 1.3. A Mach-Zehnder interferometer (top) is a sequence of two Hadamard transformations (bottom).

initely be found in the upper output. Most interestingly, this is because between the two beamsplitters the particle would have been found (with the correct relative phase) with equal probability in both beam paths. It is the interference of the two amplitudes incident on the final beamsplitter which results in the particle ending up with certainty in one of the outgoing beams and never in the other.

In quantum information language, the output qubit of the empty Mach-Zehnder interferometer will have a definite value if the input qubit also has a definite value, and this only because between the two Hadamard transformations the value of the qubit was maximally undefined.

Another important quantum gate besides the Hadamard gate is the phase shifter, which is introduced additionally in Fig. 1.4 into the Mach-Zehnder interferometer. Its operation is simply to introduce a phase change  $\varphi$  to the amplitude of one of the two beams (without loss of generality we can assume this to be the upper beam because only relative phases are relevant). In our notation, the action of the phase shifter can be described by the unitary transformation

$$\Phi|0\rangle = e^{i\varphi}|0\rangle, \quad \Phi|1\rangle = |1\rangle. \quad (1.10)$$

Therefore the output qubit can be calculated by successive application of all proper transformations to the input qubit:

$$|Q\rangle_{out} = H\Phi H|Q\rangle_{in}. \quad (1.11)$$

We leave it to the reader to calculate the general expression for arbitrary input qubits. We will restrict our discussion again to the case where we have only one input namely  $\alpha = 1$  and  $\beta = 0$ , i.e.,  $|Q\rangle_{in} = |0\rangle$ . The final state then becomes

$$H\Phi H|0\rangle = \frac{1}{2} ((e^{i\varphi} + 1)|0\rangle + (e^{i\varphi} - 1)|1\rangle). \quad (1.12)$$

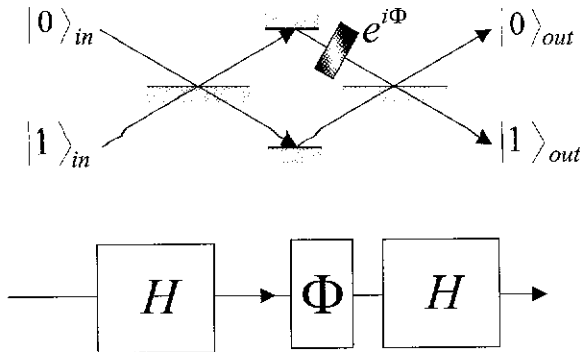


Fig. 1.4. Top: Mach-Zehnder interferometer including a phase shifter  $\varphi$  in one of the two beams. This completely changes the output. Bottom: The equivalent representation with Hadamard transformations and a phase shifter gate.

This has a very simple interpretation. First we observe by inspection of (1.12) that for  $\varphi = 0$  the value of the qubit is definitely “0”. On the other hand, for  $\varphi = \pi$  the value of the qubit is definitely “1”. This indicates that the phase shift  $\varphi$  is able to switch the output qubit between 0 and 1. In general, the probability that the output qubit has the value “0” is  $P_0 = \cos^2(\varphi/2)$ , and the probability that the qubit carries the value “1” is  $P_1 = \sin^2(\varphi/2)$ .

In the present section we have discussed some of the basic notions of linear transformation of qubits. We will now turn to entangled qubits.

## 1.4 Entanglement

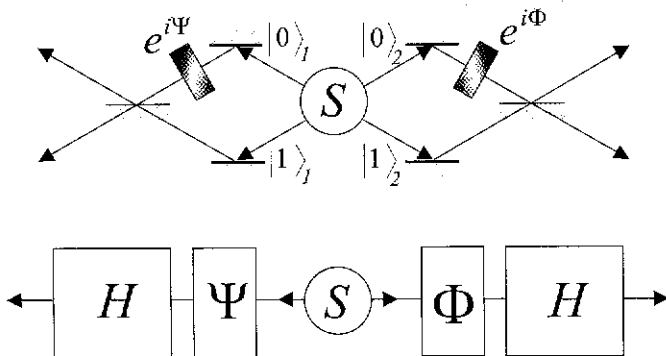
Consider a source which emits a pair of particles such that one particle emerges to the left and the other one to the right (see source S in Fig. 1.5). The source is such that the particles are emitted with opposite momenta. If the particle emerging to the left, which we call particle 1, is found in the upper beam, then particle 2 travelling to the right is always found in the lower beam. Conversely, if particle 1 is found in the lower beam, then particle 2 is always found in the upper beam. In our qubit language we would say that the two particles carry different bit values. Either particle 1 carries “0” and then particle 2 definitely carries “1”, or vice versa. Quantum mechanically this is a two-particle superposition state of the form

$$\frac{1}{\sqrt{2}} (|0\rangle_1|1\rangle_2 + e^{i\chi}|1\rangle_1|0\rangle_2) . \quad (1.13)$$

The phase  $\chi$  is just determined by the internal properties of the source and we assume for simplicity  $\chi = 0$ . Equation (1.13) describes what is called an entangled state [8]<sup>1</sup>. The interesting property is that neither of the two qubits carries a definite value, but what is known from the quantum state is that as soon as one of the two qubits is subject to a measurement, the result of this measurement being completely random, the other one will immediately be found to carry the opposite value. In a nutshell this is the conundrum of quantum non-locality, since the two qubits could be separated by arbitrary distances at the time of the measurement.

A most interesting situation arises when both qubits are subject to a phase shift and to a Hadamard transformation as shown in Fig. 1.5. Then, for detection events after both Hadamard transformations, that is, for the case of the two-particle interferometer verification [10] for detections behind the beamsplitters, interesting non-local correlations result which violate Bell’s inequalities [11]. Without going into the theoretical and formal details here (for more information see Sect. 1.7), the essence of such a violation is that

<sup>1</sup> The word *Entanglement* is a (free) translation of the word *Verschränkung* that was introduced in 1935 by Schrödinger to characterise this special feature of composite quantum systems [9].



**Fig. 1.5.** A source emits two qubits in an entangled state. Top: A two-particle interferometer verification. Bottom: The principle in terms of one-photon gates

there is no possibility to explain the correlations between the two sides on the basis of local properties of the qubits alone. The quantum correlations between the two sides cannot be understood by assuming that the specific detector on one given side which registers the particle is not influenced by the parameter setting, that is, by the choice of the phase for the other particle. There are many ways to express precisely the meaning of Bell's inequalities, and there are many formal presentations. Some of this discussion will be presented in Sect. 1.7, and for the remainder we refer the reader to the appropriate literature (e.g., Ref. [12] and references therein).

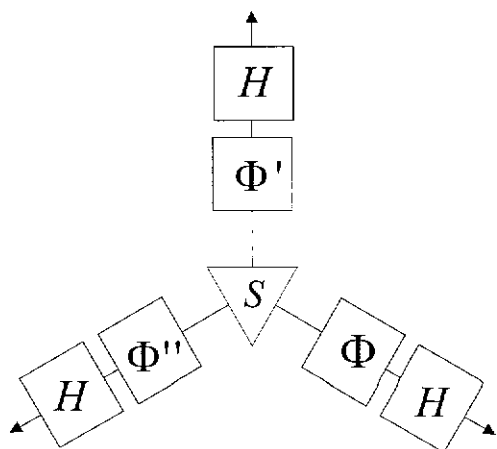
A very interesting, and for quantum computation quite relevant generalisation follows if entanglement is studied for more than two qubits. For example, consider the simple case of entanglement between three qubits, as shown in Fig. 1.6. We assume that a source emits three particles, one into each of the apparatuses shown, in the specific superposition, a so-called Greenberger–Horne–Zeilinger (GHZ) state [13] (see also Sect. 6.3),

$$\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|1\rangle_2|1\rangle_3). \quad (1.14)$$

This quantum state has some very peculiar properties. Again, as in two-particle entanglement, none of the three qubits carries any information on its own, none of them has a defined bit value. But, as soon as one of the three is measured, the other two will assume a well-defined value as long as the measurement is performed in the chosen 0-1 basis. This conclusion holds independent of the spatial separation between the three measurements.

Most interestingly, if one looks at the relations predicted by the GHZ state (1.14) between the three measurements after passing the phase shifters and the Hadamard transforms, a number of perfect correlations still result for certain joint settings of the three parameters [14], the interesting property now being that it is not possible to understand even the perfect correlations with a local model. This shows that quantum mechanics is at variance with





**Fig. 1.6.** Three-particle entanglement in a so-called GHZ state. Here we show only the representation in terms of our elementary gates, it will be straightforward for the reader to consider the physical realisation in a three-particle interferometer.

a classical local world view not only for the sector of statistical predictions of the theory but also for predictions which can be made with certainty.

## 1.5 Entanglement and Quantum Indistinguishability

In order to understand both the nature of entanglement and ways of producing it, one has to realise that in states of the general form (1.13) and (1.14), we have a superposition between product states. We recall from the discussion of the double-slit diffraction phenomenon (Sect. 1.1) that superposition means that there is no way to tell which of the two possibilities forming the superposition actually pertains. This rule must also be applied to the understanding of quantum entanglement. For example, in the state

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2) \quad (1.15)$$

there is no way of telling whether qubit 1 carries the value “0” or “1”, and likewise whether qubit 2 carries the value “0” or “1”. Yet, if one qubit is measured the other one immediately assumes a well-defined quantum state. These observations lead us directly to the conditions of how to produce and observe entangled quantum states.

To produce entangled quantum states, one has various possibilities. Firstly, one can create a source which, through its physical construction, is such that the quantum states emerging already have the indistinguishability feature discussed above. This is realised, for example, by the decay of a spin-0 particle into two spin-1/2 particles under conservation of the internal

angular momentum [15]. In this case, the two spins of the emerging particles have to be opposite, and, if no further mechanisms exist which permit us to distinguish the possibilities right at the source, the emerging quantum state is

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2), \quad (1.16)$$

where, e.g.  $|\uparrow\rangle_1$  means particle 1 with spin up. The state (1.16) has the remarkable property that it is rotationally invariant, i.e., the two spins are anti-parallel along whichever direction we choose to measure.

A second possibility is that a source might actually produce quantum states of the form of the individual components in the superposition of (1.15), but the states might still be distinguishable in some way. This happens, for example, in type-II parametric down-conversion [16] (Sect. 3.4.4), where along a certain chosen direction the two emerging photon states are

$$|H\rangle_1|V\rangle_2 \quad \text{and} \quad |V\rangle_1|H\rangle_2. \quad (1.17)$$

That means that either photon 1 is horizontally polarised and photon 2 is vertically polarised, or photon 1 is vertically polarised and photon 2 is horizontally polarised. Yet because of the different speeds of light for the  $H$  and  $V$  polarised photons inside the down-conversion crystal, the time correlation between the two photons is different in the two cases. Therefore, the two terms in (1.17) can be distinguished by a time measurement and no entangled state results because of this potential to distinguish the two cases. However, in this case too one can still produce entanglement by shifting the two photon-wave packets after their production relative to each other such that they become indistinguishable on the basis of their positions in time. What this means is the application of a quantum eraser technique [17] where a marker, in this case the relative time ordering, is erased such that we obtain quantum indistinguishability resulting in the state

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + e^{ix}|V\rangle_1|H\rangle_2), \quad (1.18)$$

which is entangled.

A third means of producing entangled states is to project a non-entangled state onto an entangled one. We remark, for example, that an entangled state is never orthogonal to any of its components. Specifically, consider a source producing the non-entangled state

$$|0\rangle_1|1\rangle_2. \quad (1.19)$$

Suppose this state is now sent through a filter described by the projection operator

$$P = |\Psi\rangle_{12}\langle\Psi|_{12}. \quad (1.20)$$

where  $|\Psi\rangle_{12}$  is the state of (1.15). Then the following entangled state results:

$$\frac{1}{2}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)(\langle 0|_1\langle 1|_2 + \langle 1|_1\langle 0|_2)|0\rangle_1|1\rangle_2 = \frac{1}{2}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2); \quad (1.21)$$

it is no longer normalised to unity because the projection procedure implies a loss of qubits.

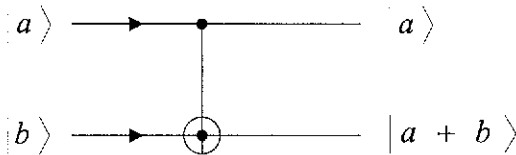
While each of the three methods discussed above can in principle be used to produce outgoing entangled states, a further possibility exists to produce entanglement upon observation of a state. In general, this means that we have an unentangled or partially entangled state of some form and the measurement procedure itself is such that it projects onto an entangled state, in much the same way as discussed just above. This procedure was used, for example, in the first experimental demonstration of GHZ entanglement of three photons (see Sect. 6.3) [18].

## 1.6 The Controlled NOT Gate

Thus far, we have discussed only single-qubit gates, that is, gates which involve one qubit only. Of greatest importance for quantum computation applications are two-qubit gates, where the evolution of one qubit is conditional upon the state of the other qubit. The simplest of these gates is the quantum controlled NOT gate illustrated in Fig. 1.7. The essence of the controlled NOT gate is that the value of the so-called target qubit is negated if and only if the control qubit has the logical value “1”. The logical value of the control qubit does not change. The action of the quantum controlled NOT gate can be described by the transformations

$$\begin{aligned} |0\rangle_c|0\rangle_t &\rightarrow |0\rangle_c|0\rangle_t & |0\rangle_c|1\rangle_t &\rightarrow |0\rangle_c|1\rangle_t \\ |1\rangle_c|0\rangle_t &\rightarrow |1\rangle_c|1\rangle_t & |1\rangle_c|1\rangle_t &\rightarrow |1\rangle_c|0\rangle_t \end{aligned} \quad (1.22)$$

where  $|0\rangle_c$  and  $|1\rangle_c$  refer to the control qubit and  $|0\rangle_t$  and  $|1\rangle_t$  refer to the target qubit. Together with the single-qubit transformations described in



**Fig. 1.7.** The controlled NOT gate is a transformation involving two qubits. The value of the control qubit (the upper one in the figure) influences the lower one, whose value is flipped if the upper qubit carries “1”, and not flipped if the upper qubit carries “0”. This is equivalent to addition modulo 2.

Sect. 1.3 the quantum controlled NOT gate can be used to realise quantum computation networks. One interesting explicit application is the production of two-qubit or multi-qubit entangled states using these gates [19].

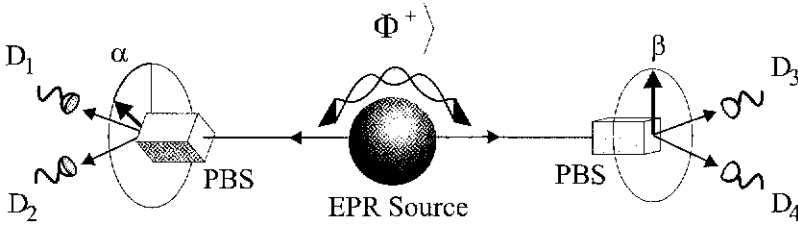
## 1.7 The EPR Argument and Bell's Inequality

Immediately after the discovery of modern quantum mechanics, it was realised that it contains novel, counterintuitive features, as witnessed most remarkably in the famous dialogue between Niels Bohr and Albert Einstein [20]. While Einstein initially tried to argue that quantum mechanics is inconsistent, he later reformulated his argument towards demonstrating that quantum mechanics is incomplete. In the seminal paper [21], Einstein, Podolsky and Rosen (EPR) consider quantum systems consisting of two particles such that, while neither position nor momentum of either particle is well defined, the sum of their positions, that is their centre of mass, and the difference of their momenta, that is their individual momenta in the center of mass system, are both precisely defined. It then follows that a measurement of either position or momentum performed on, say, particle 1 immediately implies a precise position or momentum, respectively, for particle 2, without interacting with that particle. Assuming that the two particles can be separated by arbitrary distances, EPR suggest that a measurement on particle 1 cannot have any actual influence on particle 2 (locality condition); thus the property of particle 2 must be independent of the measurement performed on particle 1. To them, it then follows that both position and momentum can simultaneously be well defined properties of a quantum system.

In his famous reply [22], Niels Bohr argues that the two particles in the EPR case are always parts of one quantum system and thus measurement on one particle changes the possible predictions that can be made for the whole system and therefore for the other particle.

While the EPR–Bohr discussion was considered for a long time to be merely philosophical, in 1951 David Bohm [15] introduced spin-entangled systems and in 1964 John Bell [23] showed that, for such entangled systems, measurements of correlated quantities should yield different results in the quantum mechanical case to those expected if one assumes that the properties of the system measured are present prior to, and independent of, the observation. Even though a number of experiments have now confirmed the quantum predictions [24]–[26], from a strictly logical point of view the problem is not closed yet as some loopholes in the existing experiments still make it logically possible, at least in principle, to uphold a local realist world view [27].

Let us briefly present the line of reasoning that leads to an inequality equivalent to the original Bell inequality. Consider a source emitting two qubits (Fig. 1.8) in the entangled state



**Fig. 1.8.** Correlation measurements between Alice's and Bob's detection events for different choices for the detection bases (indicated by the angles  $\alpha$  and  $\beta$  for the orientation of their polarising beamsplitters, PBS) lead to the violation of Bell's inequalities.

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2). \quad (1.23)$$

One qubit is sent to Alice (to the left in Fig. 1.8), the other one to Bob (to the right). Alice and Bob will perform polarisation measurements using a polarising beamsplitter with two single-photon detectors in the output ports. Alice will obtain the measurement result “0” or “1”, corresponding to the detection of a qubit by detector 1 or 2 respectively, each with equal probability. This statement is valid in whatever polarisation basis she decides to perform the measurement, the actual results being completely random. Yet, if Bob chooses the same basis, he will always obtain the same result. Thus, following the first step of the EPR reasoning, Alice can predict with certainty what Bob's result will be. The second step employs the locality hypothesis, that is, the assumption that no physical influence can instantly go from Alice's apparatus to Bob's, and therefore Bob's measured result should only depend on the properties of his qubit and on the apparatus he chose. Combining the two steps, John Bell investigated possible correlations for the case that Alice and Bob choose detection bases which are at oblique angles. For three arbitrary angular orientations  $\alpha$ ,  $\beta$ ,  $\gamma$ , one can see [28] that the following inequality must be fulfilled:

$$N(1_\alpha, 1_\beta) \leq N(1_\alpha, 1_\gamma) + N(1_\beta, 0_\gamma), \quad (1.24)$$

where

$$N(1_\alpha, 1_\beta) = \frac{N_0}{2} \cos^2(\alpha - \beta) \quad (1.25)$$

is the quantum-mechanical prediction for the number of cases where Alice obtains “1” with her apparatus at orientation  $\alpha$  and Bob achieves “1” with orientation  $\beta$ , and  $N_0$  is the number of pairs emitted by the source. The inequality is violated by the quantum-mechanical prediction if we choose, for example, the angles  $(\alpha - \beta) = (\beta - \gamma) = 30^\circ$ . The violation implies that at least one of the assumptions entering Bell's inequality must be in conflict with quantum mechanics. This is usually viewed as evidence for non-locality, though that is by no means the only possible explanation.

## 1.8 Comments

As recently as a decade ago, the issues discussed here were mainly considered to be of a philosophical nature, though very relevant ones in our attempts to understand the world around us and our role in it. In the last few years, very much to the surprise of most of the early researchers in the field, the basic concepts of superposition and quantum entanglement have turned out to be key ingredients in novel quantum communication and quantum computation schemes. Here we have given only a condensed introduction. More details are contained in the various chapters of this book. Further information can also be found on the world wide web, for example at [www.qubit.org](http://www.qubit.org) or [www.quantum.at](http://www.quantum.at) with many links to other relevant sites.